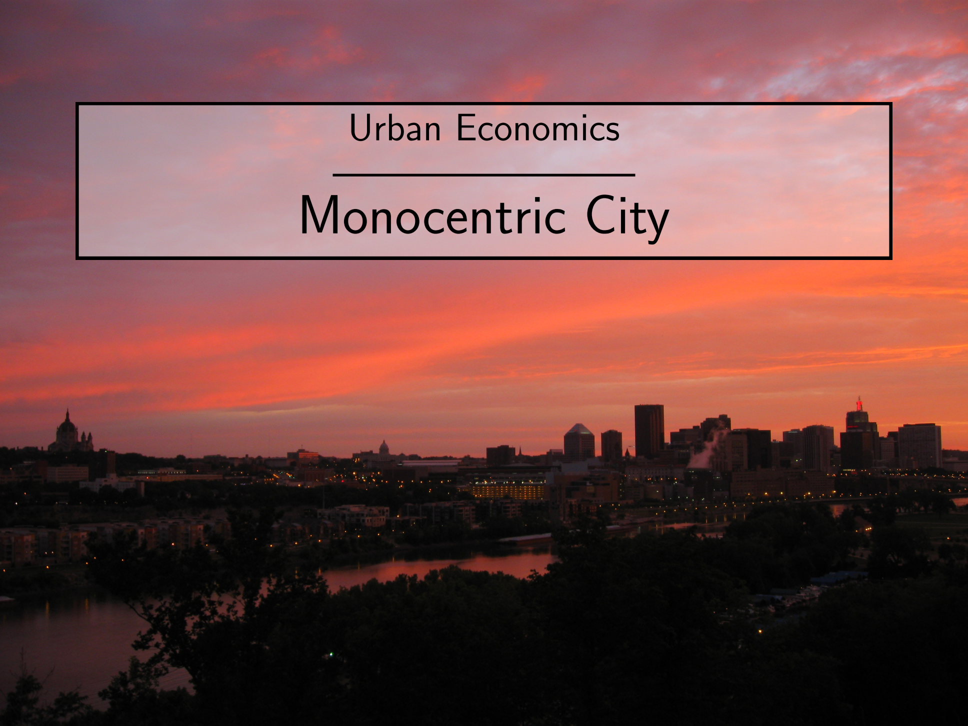
Urban Economics

Monocentric City

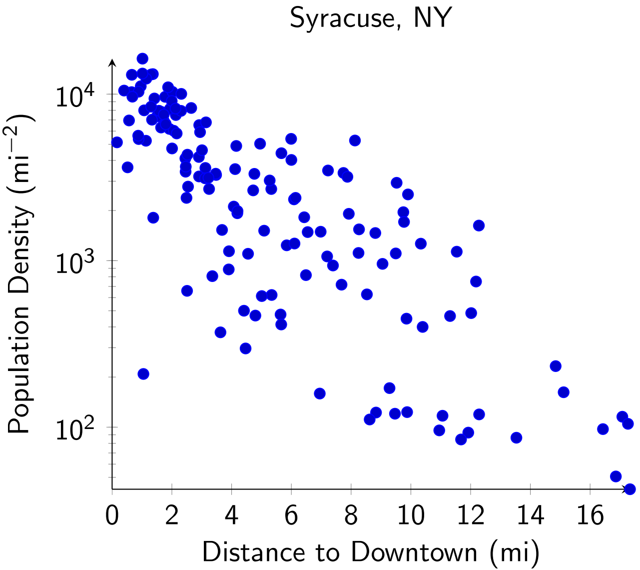


Land Price and Location

> "Three things matter in real estate: location, location, location."

- Compare prices for similarly sized houses close to and away from downtown.

Population Density and Location



Population density in Onondaga County in 2000. Each dot represents a census tract.

Monocentric City Model

- Monocentric city model will explain observed patterns of land prices and population densities with little more than transportation costs.
- Variations and minor extensions will be able to address the land use and the structure of cities, segregation, transportation policy, and more.
- The model has consumer utility in it, which allows for welfare analysis.

Residents in the Monocentric City Model

Residents are the main agents in this model.

Residents all:

- work earning income y
- commute to the city center at a cost tx
  - t is a transportation cost
  - x is how far the resident lives from the city center
  - residents may have different xs
  - all residents are assumed to work in the city center (hence "monocentric city")
- have preferences for land (q) and nonland consumption (c) given by a utility function $u(q, c)$

Residents' Budget Constraint

Let p be the price of land.
Let the price of nonland consumption set to 1.

Budget constraint:

$$\underbrace{pq}_{\substack{\text{expenditures}\\\text{on land}}} + \underbrace{c}_{\substack{\text{other}\\\text{expenditures}}} = \underbrace{y}_{\text{income}} - \underbrace{tx}_{\substack{\text{commuting}\\\text{cost}}}$$

Residents' Budget Constraint

Let p be the price of land.
Let the price of nonland consumption set to 1.

Budget constraint:

$$pq + c = y - tx \tag{BC}$$

Residents' Problems

Given prices, income, and parameters, residents chose their location x and the consumption amounts q and c to maximize utility.

$$\max_{x,q,c} u(q,c) \text{ subject to } pq + c = y - tx$$

- The utility attainable at any location will be the same in equilibrium.
  - Otherwise, everyone would move from the worst location to the best.

Other model elements

H	fixed number of residents
p_{ag}	agricultural value of land
$\hat{x}$	location of the boundary between workers and agricultural land

Equilibrium

An equilibrium for this model is a city boundary $\hat{x}$, a land price $p^*(x)$ defined, and consumption quantities $q^*(x)$ and $c^*(x)$ that satisfy the following conditions:

1. $q^*(x)$ and $c^*(x)$ maximize $u(q, c)$ subject to $pq + c = y - tx$.
2. residents are indifferent to any location between 0 and $\hat{x}$.

   $$u\left(q^*(x_1)c^*(x_1)\right) = u\left(q^*(x_2)c^*(x_2)\right) \text{ for all } x_1, x_2 \in [0, \hat{x}]$$

3. $p(\hat{x}) = p_{ag}$
4. land supply equals demand

   $$\int_0^{\hat{x}} \frac{1}{q(x)} dx = H$$

Equilibrium with Calibrated Cobb-Douglas Utility

Let $u(q, c) = q \cdot c^2$.

Solution outline:

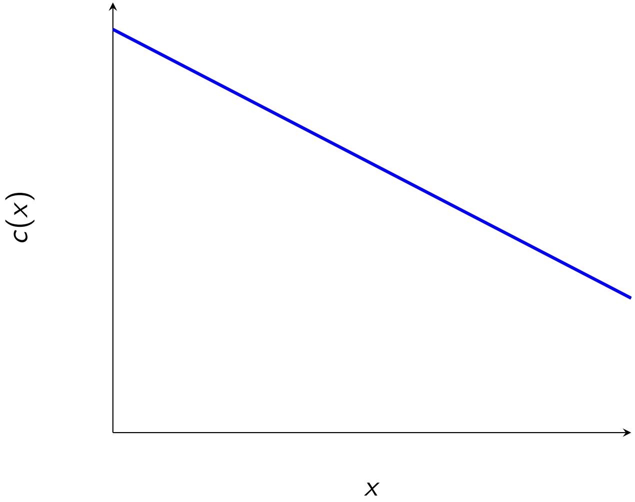
1. Solve the residents problem at each x:

$$\max_{q,c} q \cdot c \text{ subject to } pq + c = y - tx$$

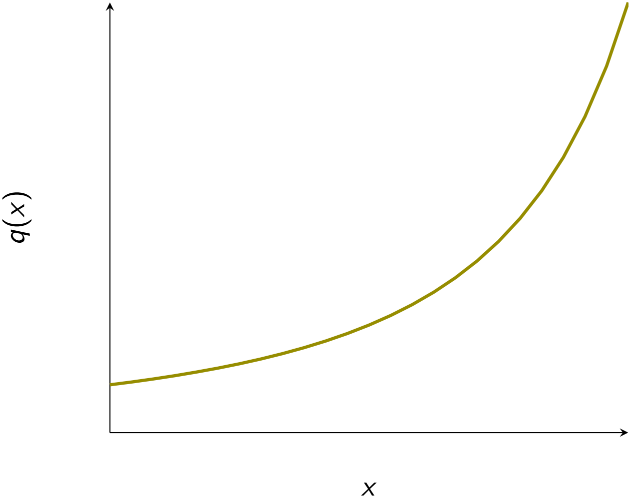
   Yields q^* and c^* as functions of x and $p^*(x)$.

2. Use indifference between locations to find $p^*(x)$ in terms of x and unknown constants.
3. Use the last equilibrium conditions to solve for the constants. Yields q^* and c^* as functions of x, t, y, H, p_{ag}.

Nonland consumption



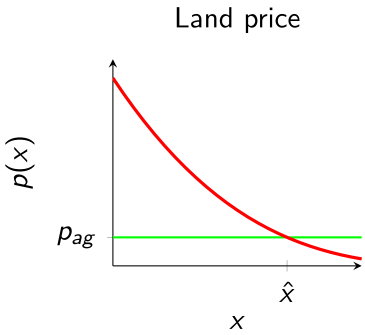
Land consumption

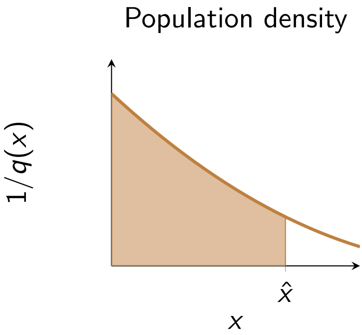


Population density

Land price

Finding the City Boundary





The two final equilibrium conditions:

- $p(\hat{x}) = p_{ag}$
- $\int_0^{\hat{x}} \frac{1}{q(x)} dx = H$

Equilibrium with Quasilinear Utility

Now, let $u(q, c) = c + \sqrt{q}$.

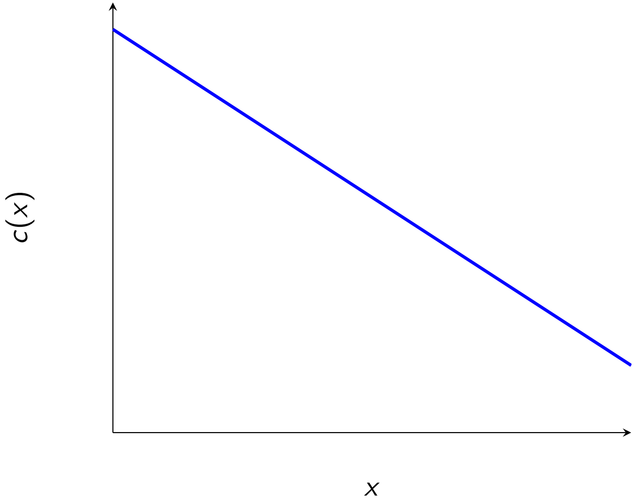
Solution outline:

1. Solve the residents problem at each x:

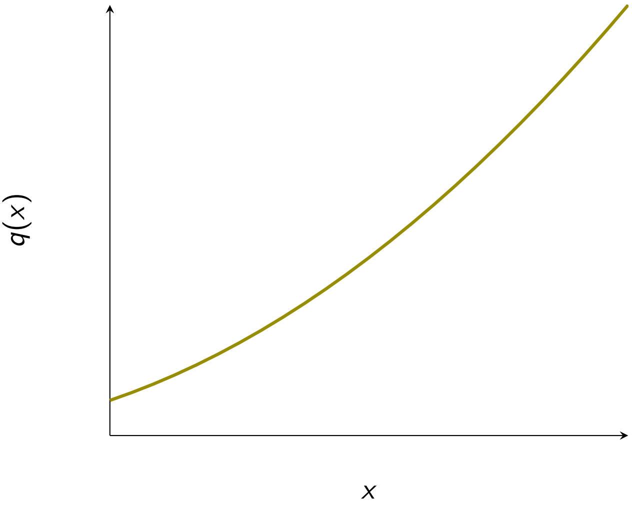
   $$\max_{q,c} c + \sqrt{q} \text{ subject to } pq + c = y - tx$$

   Yields q^* as a function of x and $p^*(x)$.
2. Use indifference between locations to find $p^*(x)$ in terms of x and unknown constants.
3. Use the last equilibrium conditions to solve for the constants. Yields q^* and c^* as functions of x, t, y, H, p_{ag}.

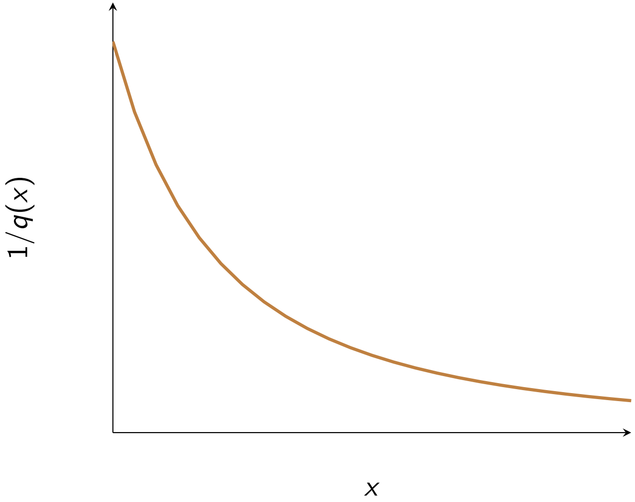
Nonland consumption



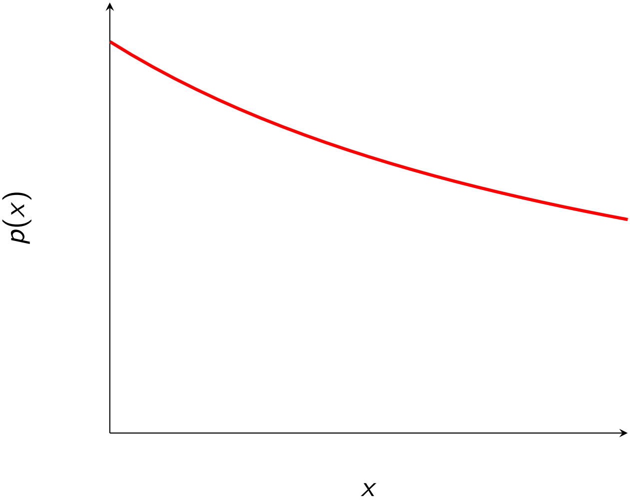
Land consumption



Population density



Land price



Results so far

With either utility function:

- land prices are less further from the city
- cities are more dense near the center
- suburban residents use more land and consume less of everything else

The shapes of the population density and land prices curves differ.

- Let us find which results will hold in general (and not merely under specific utility functions).

Land prices higher near center

From the budget constraint:

$$pq + c = \underbrace{y - tx}_{\text{decreasing in x}}$$

- the utility level $u\left(c^*(x), q^*(x)\right)$ is the same at every location
- if $p(x)$ was constant, then suburban residents attain the same utility with less expenditure
- as long as utility is increasing in q (and non-decreasing in c), then equilibrium $p(x)$ is decreasing in x

Land Price and Use

Using only the budget constraint, the condition that utility must be constant in x, and an assumption that u is differentiable, it can be shown:

$$\frac{dp^*(x)}{dx} = \frac{-t}{q(x)}$$

One implication of this:

$$q(x) > 0 \implies \frac{-t}{q(x)} < 0 \implies \frac{dp^*(x)}{dx} < 0 \implies p^* \text{ is decreasing in } x$$

(just as argued previously)

Smaller land holdings, higher density near center

- $c^*(x)$ and $q^*(x)$ cannot both be decreasing in x, otherwise $u(c^*(x), q^*(x))$ would not be the same everywhere.
- Since $p(x)$ is decreasing in x, the opportunity cost of land is lower in suburbs (that is, for large x).
- In any well-behaved utility function, the result is $q^*(x)$ increasing in x.
- Population density is $1/q^*(x)$, so density is decreasing in x.

One More Utility Function

$$u(c, q) = \begin{cases} -\infty & \text{if } q < 1 \\ c & \text{if } q \geq 1 \end{cases}$$

$q = 1$ as long as $p \leq y - tx$

Residents' problem:

$$\max c \qquad \text{subject to } c + p(x) \cdot 1 = y - tx$$

Solution: $c = y - tx - p(x)$

Deriving Land Prices under Threshold Utility

$u(c, q)$ the same everywhere implies c is the same near city center.

$$\begin{aligned} \bar{c} &= y - tx - p(x) \\ p(x) &= y - tx - \bar{c} \end{aligned}$$

All H residents buy 1 unit of land, so the city boundary is at $\hat{x} = H$.

$$\begin{aligned} p(H) &= p_{ag} \\ y - tH - \bar{c} &= p_{ag} \\ \bar{c} &= y - tH - p_{ag} \end{aligned}$$

$$\begin{aligned} p(x) &= y - tx - \bar{c} \\ p(x) &= y - tx - (y - tH - p_{ag}) \\ p(x) &= t(H - x) + p_{ag} \end{aligned}$$

Model and Reality

- The monocentric city model matches the observed patterns for land prices and population density.
- Commuting costs drive both patterns.
- This simple model has utility, so welfare statements are possible.